

# Relating Neutrino Masses to dilepton modes of Doubly Charged Scalars

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## Abstract

We study a model with Majorana neutrino masses generated through doubly charged scalars at two-loop level. We give explicit relationships between the neutrino masses and the same sign dilepton decays of the doubly charged scalars. In particular, we demonstrate that in the tribimaximal limit of the neutrino mixings, the absolute neutrino masses and Majorana phases can be extracted through the measurements of the dilepton modes at colliders.

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## I. INTRODUCTION

It has been revealed that at least two light neutrinos have nonzero masses and the mixing matrix is characterized by two large mixing angles from neutrino oscillation experiments. These evidences exhibit new physics beyond the standard model (SM). However, the origin of neutrino masses is still mysterious even though considerable efforts have been put in both theory and experiment for decades. In particular, two crucial properties of neutrinos, which can not be disclosed in the neutrino oscillation experiments, are the Majorana nature of neutrinos and the absolute values of the neutrino masses. Presently, the only neutrino experiment that could provide a direct evidence of Majorana neutrinos is the search for the neutrinoless double beta ( $0\nu\beta\beta$ ) decays, which could be used to set some limits on the absolute neutrino masses [1]. The absolute neutrino mass can be also measured through the electron energy spectrum away from the end-point in the nuclear beta decay (i.e. the tritium decay,  $m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_{\nu_i}^2} < 2$  eV) [2, 3], while the sum of the neutrino masses has been constrained from cosmology, given by  $\sum_i m_{\nu_i} < 0.58$  eV (95% CL) [4].

Instead of probing neutrino properties at low energy experiments, the possibility to explore them at the Large Hadron Collider (LHC) has been widely studied in the literature. There are three different types of seesaw mechanisms to generate neutrino masses at tree level. The “minimal” type-I seesaw mechanism [5] can not be directly tested at collider experiments due to the suppressed mixings between the light neutrinos and heavy right-handed singlets unless some symmetry is introduced [6–9]. However, the unsuppressed gauge interactions [12, 13] of both scalar and fermionic triplets will help us to test the ideas of type-II [10] and type-III [11] seesaw mechanisms at the LHC, respectively. Furthermore, an one to one correspondence between dilepton decay widths and neutrino masses exists in the type-II seesaw model where the triplet scalar  $T$  couples to the left-handed lepton doublets  $\ell_L$  via the gauge invariant Yukawa interaction:

$$L = \bar{\ell}_L^c h_{ab} i\tau_2 T \ell_{bL} + h.c. , \quad (1)$$

where the indices  $a, b$  denote  $e, \mu, \tau$  and  $\ell_{jL}$  represents the left-handed lepton doublet of the  $j$ th flavor. The neutrino mass matrix is given by

$$m_{\nu ab} = \sqrt{2} h_{ab} v_T \quad (2)$$

after the triplet receives the vacuum expectation value (VEV),  $\langle T \rangle = v_T / \sqrt{2}$ . The decay

width of each same sign dilepton decay mode of the doubly charged Higgs scalar  $P^{\pm\pm}$ ,  $\Gamma(P^{\pm\pm} \rightarrow \ell_{aL}^{\pm} \ell_{bL}^{\pm})$ , is directly related to the corresponding neutrino mass matrix element through Eq. (2). Thus, the discovery of  $P^{\pm\pm}$  may help us to understand the Majorana nature of neutrinos, and by studying the branching ratios of the dilepton channels we may obtain some important informations such as the absolute neutrino masses and Majorana phases. The related phenomenologies have been extensively studied in the literature [14].

In this work, we study a model originally proposed in Ref. [15], in which a discrete symmetry is imposed to forbid the Yukawa coupling in Eq. (1) at tree level. As a result, the neutrino masses are generated at two-loop level with the normal hierarchy spectrum [16], while the  $0\nu\beta\beta$  decay arises at tree level [15, 17]. One of the interesting properties of the model is that it shares the same feature of the direct link between the decay widths of the same sign dilepton modes and neutrino masses as the type-II seesaw model. We will focus on the parameters which can not be measured in the neutrino oscillation experiments such as the absolute masses of three light neutrinos and the Majorana phases  $\psi_1$  and  $\psi_2$ . By assuming the tribimaximal mixings [18] and utilizing the measured mass square differences for the neutrinos, we derive some explicit relations between the neutrino masses and the branching fractions of the doubly charged scalars to the charged lepton pairs. In addition, as the neutrino masses are proportional to the products of the charged lepton masses  $m_a m_b$  due to the loop integral, the fractions  $\frac{BR(P^{\pm\pm} \rightarrow e^{\pm} \mu^{\pm})}{BR(P^{\pm\pm} \rightarrow e^{\pm} \tau^{\pm})} = \frac{m_e^2}{m_\mu^2}$  and  $\frac{BR(P^{\pm\pm} \rightarrow \mu^{\pm} \mu^{\pm})}{BR(P^{\pm\pm} \rightarrow \tau^{\pm} \tau^{\pm})} = \frac{m_\tau^4}{m_\mu^4}$  are much larger than those of unity predicted in the type-II seesaw model in the limit of the tribimaximal mixings. Clearly, we are able to differentiate the two types of the models by counting the events arising from the dilepton decays of the doubly charged Higgs scalars at the LHC.

Our paper is organized as follows. In Sec. II, we briefly introduce the model. In Sec. III, we relate the neutrino masses with the dilepton modes of the doubly charged scalars. We conclude our results in Sec. IV.

## II. THE MODEL

The simplest version of the model has been given in Ref. [15] with some of its phenomenologies presented in Ref. [16]. The idea of the model is to suppress the Yukawa interaction in Eq. (1) at tree level and induce it radiatively. Here, we give an explicit exam-

ple<sup>1</sup> to forbid the tree contributions to the neutrino masses. The model consists two Higgs doublets  $\phi_1$  and  $\phi_2$ , one triplet  $T$  and one doubly charged singlet  $\Psi$  with the hypercharges of  $-1/2$ ,  $-1$  and  $2$ , respectively. Besides the gauge symmetry, there is a  $Z_2$  discrete symmetry with the transformations of  $\phi_1 \rightarrow +\phi_1$ ,  $\phi_2 \rightarrow -\phi_2$ ,  $T \rightarrow -T$ ,  $\Psi \rightarrow +\Psi$ , and  $f \rightarrow f$ , where  $f$  represents the SM fermion. The most general potential in this model can be written as

$$\begin{aligned}
V = & -\mu_1^2 \phi_1^\dagger \phi_1 + \lambda_1 (\phi_1^\dagger \phi_1)^2 - \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\
& -\mu_T^2 \text{Tr}(T^\dagger T) + \lambda_T [\text{Tr}(T^\dagger T)]^2 + \lambda'_T \text{Tr}(T^\dagger T T^\dagger T) \\
& + m^2 \Psi^\dagger \Psi + \lambda_\Psi (\Psi^\dagger \Psi)^2 \\
& + \kappa_{\phi_1} \text{Tr}(\phi_1^\dagger \phi_1 T^\dagger T) + \kappa'_{\phi_1} \phi_1^\dagger T T^\dagger \phi_1 + \kappa_{\Psi_1} \phi_1^\dagger \phi_1 \Psi^\dagger \Psi \\
& + \kappa_{\phi_2} \text{Tr}(\phi_2^\dagger \phi_2 T^\dagger T) + \kappa'_{\phi_2} \phi_2^\dagger T T^\dagger \phi_2 + \kappa_{\Psi_2} \phi_2^\dagger \phi_2 \Psi^\dagger \Psi \\
& + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 + \rho \text{Tr}(T^\dagger T \Psi^\dagger \Psi) \\
& + (M \phi_1^T T^\dagger \phi_2 + \lambda_5 \phi_1^\dagger \phi_2 \phi_1^\dagger \phi_2 + \lambda \tilde{\phi}_1^\dagger T \tilde{\phi}_2^* \Psi + \text{H.c.}), \tag{3}
\end{aligned}$$

where  $\tilde{\phi}_i = i\tau_2 \phi_i^*$ . Under the symmetries, the doubly charged singlet  $\Psi$  couples to the right-handed charged leptons  $l_R$  via the Yukawa interaction,

$$L_Y = Y_{ab} \bar{\ell}_c^a \ell_{bR} \Psi + \text{h.c.}, \tag{4}$$

where  $Y_{ab}$  is a  $3 \times 3$  symmetric matrix with the indices  $a, b$  stand for  $e, \mu, \tau$  and  $\ell_{jR}$  represent the right-handed lepton singlets. We note that the interaction between the scalar triplet and the left-handed lepton doublets is not allowed by imposing the  $Z_2$  symmetry such that there is no neutrino mass term at tree level unlike the type-II seesaw mechanism. When the scalar fields  $\phi_{1,2}$  and  $T$  develop VEVs, both the gauge and  $Z_2$  discrete symmetries spontaneously break down. The neutrino masses will be generated through two-loop diagrams as shown in Ref. [15], given by

$$(m_\nu)_{ab} = \frac{g^4 v_T Y_{ab} \sin 2\theta}{\sqrt{2}} m_a m_b [I(M_{P_1}^2) - I(M_{P_2}^2)], \tag{5}$$

where  $v_T < 4$  GeV is the VEV of the scalar triplet  $T$ , bounded by the  $\rho$ -parameter ( $= M_W^2/M_Z^2 \cos^2 \theta_W$ ) [2, 19],  $m_{a,b}$  stand for the charged lepton masses,  $M_{P_{1,2}}$  are the masses of the doubly charged scalar eigenstates  $P_{1,2}^{\pm\pm}$  with the mixing angle  $\theta$  defined by

$$\begin{pmatrix} P_1^{\pm\pm} \\ P_2^{\pm\pm} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} T^{\pm\pm} \\ \Psi^{\pm\pm} \end{pmatrix}, \tag{6}$$

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<sup>1</sup> Two possible scenarios have been mentioned in the footnote of Ref. [15].

and the integral  $I(M_{P_i}^2)$  is expressed as

$$I(M_{P_i}^2) = \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m_a^2} \frac{1}{k^2 - M_W^2} \frac{1}{q^2 - M_W^2} \frac{1}{q^2 - m_b^2} \frac{1}{(k - q)^2 - m_{P_i}^2}, \quad (7)$$

which can be approximated to be  $I \sim \frac{1}{(4\pi)^4 M_{P_i}^2} \log^2 \left( \frac{M_W^2}{M_{P_i}^2} \right)$  for  $M_{P_i} > M_W$  [20]. From Eq. (5), we see that the one to one correspondence between neutrino mass elements  $m_{\nu_{ab}}$  and the Yukawa couplings  $Y_{ab}$  provides us the opportunity to determine the neutrino masses through the measurements of the dilepton modes. We note that even for small neutrino masses, the  $0\nu\beta\beta$  decays in this model can be large since they are dominated by the exchanges of the doubly charged scalars at tree level [15].

### III. DECAY BRANCHING RATIOS RELATED TO NEUTRINO MASSES

#### A. Dilepton decays

There are several channels that the doubly charged scalars can decay into, such as  $P^{\pm\pm} \rightarrow \ell_{aR}^\pm \ell_{bR}^\pm, W^\pm W^\pm, W^\pm P^\pm, P^\pm P^\pm$  and  $W^\pm W^\pm P^0$ , where  $P^{\pm\pm}$  are referred to the lighter mass eigenstate among  $P_i^{\pm\pm}$  ( $i = 1, 2$ ). Due to the kinematical consideration, we expect that the first two kinds of the modes contribute the most part to the width of  $P^{\pm\pm}$  if  $M_{P^{\pm\pm}} \sim M_{P^\pm} \sim M_{P^0}$ . However, as long as the mass splitting between  $P^{\pm\pm}$  and  $P^\pm(M_{P^0})$  is large enough, the last three channels may dominate at the high mass region. In our discussion, we assume that the last three types of decay channels with scalar(s) in the final states are not allowed. Hence, the two-loop suppression factor in neutrino masses of Eq. (5) makes the fraction of  $\frac{\Gamma(l_a R l_b R)}{\Gamma(WW)} \approx \frac{16(4\pi)^8}{g^{12}} \left( \frac{|m_{\nu_{ab}}| M_{P_1}}{m_a m_b} \right)^2 \left( \frac{M_W}{v_T} \right)^4 \sim \mathcal{O}(10^{20})$ , so that the branching ratio with the dilepton final states is almost 100% for  $v_T \lesssim \mathcal{O}(1)$  GeV by taking  $m_{\nu_{ab}} = 0.1$  eV and  $M_{P_1} = 200$  GeV. While the fraction in the seesaw type-II model is  $(\frac{m_{\nu_{ab}}}{M_{H^{\pm\pm}}})^2 (\frac{M_W}{v_T})^4 \gtrsim 1$  only for  $v_T \lesssim \mathcal{O}(10^{-4})$  GeV with the same input. We take the values of  $m_{\nu_{ee}} \sim 0.01$  eV,  $m_{\nu_{e\mu}} \sim m_{\nu_{e\tau}} \sim 0.1$  eV, and  $m_{\nu_{\mu\mu}} \sim m_{\nu_{\mu\tau}} \sim m_{\nu_{\tau\tau}} \sim 1$  eV to illustrate the possible branching ratios of the dilepton modes in our model. These values are based on the texture of the neutrino mass matrix with the normal hierarchical spectrum as predicted by this model [16]. The estimations of the decays for  $M_P = 200$  GeV and  $v_T = 1$  GeV are shown in Table I. One should keep in mind that there might exist some cancellations among the combinations of three neutrino masses and Majorana phases in each element of the neutrino mass matrix.

For example, the component  $m_{\nu_{ee}}$  may go to zero with certain values of Majoran phases and the lightest neutrino mass for the normal hierarchical spectrum. In this case,  $0\nu\beta\beta$  decays may be out of the reach of the current experimental sensitivity. Therefore, the branching ratios of the dilepton modes shown in Table I will change drastically with different values of Majorana phases and neutrino masses, which will be discussed in Sec. III C.

The production of  $P^{\pm\pm}$  is dominated by the Drell-Yan process, for which the next to leading-order contribution from QCD enhances the cross section by a factor of 1.25 at the LHC [21]. Several simulations have been performed for  $P^{\pm\pm} \rightarrow \ell_a^\pm \ell_b^\pm$  at the LHC [22]. For channels of the doubly charged Higgs decays into  $e$  or  $\mu$ , the SM background is shown to be negligible in the signal region of the high invariant mass close to  $M_{P^{\pm\pm}}$ . In contrast, the spectrum of the missing transverse momentum for the  $\tau$  decay product will be softer due to the subsequent decays of  $\tau \rightarrow e\nu\bar{\nu}$  and  $\tau \rightarrow \mu\nu\bar{\nu}$  with a branching ratio around 17% in each channel. Additional backgrounds with jets, such as  $W^\pm W^\pm jj$ , may fake the hadronic  $\tau$  decays. As a result, the  $\tau$  tag efficiency is about 50% and the fake rate is around 1% [23]. The detector-specific error analysis is beyond the scope of this paper. We have used uniform uncertainties for all branching ratios for the rough estimation of the effect. For example, the approximate event numbers for the pair production of  $P^{\pm\pm}$  are 15000, 3000 and 900 with  $M_P = 200, 300$  and  $400$  GeV, respectively, at the LHC ( $\sqrt{s} = 14$  TeV) for the integrated luminosity of  $L = 300 fb^{-1}$ ,  $BR(P^{\pm\pm} \rightarrow \ell_a^\pm \ell_b^\pm) = 100\%$ , and the efficiency  $\epsilon_{eff} = 0.5$ . We write the dilepton decay widths as

$$\Gamma(P^{\pm\pm} \rightarrow \ell_{aR}^\pm \ell_{bR}^\pm) = \frac{|Y_{ab}|^2}{8\pi(1 + \delta_{ab})} s_\theta^2 M_P. \quad (8)$$

TABLE I. The branching ratios of  $P^{\pm\pm} \rightarrow \ell_a^\pm \ell_b^\pm$  ( $\ell_{a,b}^\pm = e^\pm, \mu^\pm$  and  $\tau^\pm$ ) by assuming the neutrino mass elements of  $m_{\nu_{ee}} = 0.01$ ,  $m_{\nu_{e\mu}} = m_{\nu_{e\tau}} = 0.1$ , and  $m_{\nu_{\mu\mu}} = m_{\nu_{\mu\tau}} = m_{\nu_{\tau\tau}} = 1$  eV for  $v_T = 1$  GeV and  $M_P = 200$  GeV [16].

$BR_{e^\pm e^\pm}$	$BR_{e^\pm \mu^\pm}$	$BR_{e^\pm \tau^\pm}$	$BR_{\mu^\pm \mu^\pm}$	$BR_{\mu^\pm \tau^\pm}$	$BR_{\tau^\pm \tau^\pm}$
0.995	$4.6 \times 10^{-3}$	$1.6 \times 10^{-5}$	$5.4 \times 10^{-6}$	$3.8 \times 10^{-8}$	$6.7 \times 10^{-11}$

## B. Neutrino masses and mixings

Since the decay widths in Eq. (8) are proportional to  $Y_{ab}$ , which are related to the neutrino masses in Eq. (5) it provides with us an opportunity to study the spectrum of neutrinos at the LHC. We can express the branching ratios of the same sign charged lepton pair modes in terms of the components of the neutrino mass matrix

$$BR_{ab} = \frac{\Gamma(\ell_{aR}^\pm \ell_{bR}^\pm)}{\Gamma_{total}} = \frac{\sin_\theta^2 M_{P1}}{\Gamma_{total} \times 4\pi g^8 v_T^2 \sin^2 2\theta [I(M_{P1}^2) - I(M_{P2}^2)]} \times \frac{|m_{\nu_{ab}}|^2}{(1 + \delta_{ab})m_a^2 m_b^2} \propto \frac{|m_{\nu_{ab}}|^2}{(1 + \delta_{ab})m_a^2 m_b^2}. \quad (9)$$

It will be clear later that the overall factor including the total decay width is irrelevant and the dependence of charged lepton masses appearing in the dilepton branching ratios is due to the loop integral in our model.

Similar to the CKM mixing matrix in the quark sector, the neutrino mass matrix can be diagonalized by the unitary matrix  $U_{PMNS}$ , defined by [24],

$$M_\nu = U_{PMNS} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 e^{i\psi_1} & 0 \\ 0 & 0 & m_3 e^{i\psi_2} \end{pmatrix} U_{PMNS}^T, \quad (10)$$

where  $\psi_{1,2}$  are referred to as the Majorana phases and the PMNS matrix can be parametrized as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (11)$$

with the Dirac phase of  $\delta$ . Currently, the neutrino oscillation experiments can only give the mass-square differences [2]:

$$\Delta m_{21}^2 = (7.59 \pm 0.20) \times 10^{-5} \text{eV}^2, \quad |\Delta m_{32}^2| = (2.43 \pm 0.13) \times 10^{-3} \text{eV}^2, \quad (12)$$

and the mixing angles [2]:

$$\sin^2(2\theta_{12}) = 0.87 \pm 0.03, \quad \sin^2(2\theta_{23}) \simeq 1, \quad \sin^2(2\theta_{13}) < 0.19. \quad (13)$$

A very good approximation of leptonic mixing matrix is proposed with the so-called tribi-maximal mixing form [18]

$$\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0. \quad (14)$$

In this limit, we can express the elements of the neutrino mass matrix as

$$\begin{aligned}
|m_{\nu_{ee}}|^2 &= \frac{4}{9}m_1^2 + \frac{4}{9}m_1m_2 \cos \psi_1 + \frac{1}{9}m_2^2, \\
|m_{\nu_{e\mu}}|^2 &= \frac{1}{9}m_1^2 - \frac{2}{9}m_1m_2 \cos \psi_1 + \frac{1}{9}m_2^2, \\
|m_{\nu_{e\tau}}|^2 &= |m_{\nu_{e\mu}}|^2, \\
|m_{\nu_{\mu\mu}}|^2 &= \frac{1}{36}m_1^2 + \frac{1}{9}m_2^2 + \frac{1}{4}m_3^2 + \frac{1}{9}m_1m_2 \cos \psi_1 + \frac{1}{6}m_1m_3 \cos \psi_2 + \frac{1}{3}m_2m_3 \cos (\psi_1 - \psi_2), \\
|m_{\nu_{\mu\tau}}|^2 &= \frac{1}{36}m_1^2 + \frac{1}{9}m_2^2 + \frac{1}{4}m_3^2 + \frac{1}{9}m_1m_2 \cos \psi_1 - \frac{1}{6}m_1m_3 \cos \psi_2 - \frac{1}{3}m_2m_3 \cos (\psi_1 - \psi_2), \\
|m_{\nu_{\tau\tau}}|^2 &= |m_{\nu_{\mu\mu}}|^2.
\end{aligned} \tag{15}$$

### C. Relations

Since the normal hierarchical mass spectrum of the light neutrinos is predicted in our model [16], we can parametrize the eigenvalues of the masses in terms of the mass differences and the lightest one  $m_1$ , given by

$$m_1, \quad m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2}. \tag{16}$$

By utilizing Eqs. (9) and (15), we can define the quantity

$$C_1 = \frac{m_\mu^2(2m_\mu^2 BR_{\mu\mu} + m_\tau^2 BR_{\mu\tau} + m_e^2 BR_{e\mu})}{2m_e^2(m_e^2 BR_{ee} + m_\mu^2 BR_{e\mu})} = \frac{m_1^2 + \frac{5}{6}\Delta m_{21}^2 + \frac{1}{2}\Delta m_{32}^2}{m_1^2 + \frac{1}{3}\Delta m_{21}^2} \tag{17}$$

such that we can determine the lightest neutrino mass  $m_1$  via the quantity  $C_1$  by measuring the branching ratios of the  $ee, e\mu, \mu\mu$  and  $\mu\tau$  channels with the relation

$$m_1^2 = \frac{(\frac{5}{6} - \frac{1}{3}C_1)\Delta m_{21}^2 + \frac{1}{2}\Delta m_{32}^2}{C_1 - 1}. \tag{18}$$

Similarly, we can express the masses  $m_2$  and  $m_3$  as

$$m_2^2 = \frac{(C_2 + 2)\Delta m_{21}^2 + 3C_2\Delta m_{32}^2}{3(1 - C_2)} \tag{19}$$

and

$$m_3^2 = \frac{(9 - 8C_3)\Delta m_{32}^2 + (6 - C_3)\Delta m_{21}^2}{9 - 17C_3}, \tag{20}$$

with the quantities  $C_2$  and  $C_3$ , defined by

$$C_2 = \frac{m_e^2(m_e^2 BR_{ee} + m_\mu^2 BR_{e\mu})}{2m_\mu^4 BR_{\mu\mu} + m_\mu^2 m_\tau^2 BR_{\mu\tau} - m_e^4 BR_{ee}} \tag{21}$$



and

$$C_3 = \frac{m_\mu^2(2m_\mu^2 BR_{\mu\mu} + m_\tau^2 BR_{\mu\tau}) + m_e^2(2m_\mu^2 BR_{e\mu} - m_e^2 BR_{ee})}{m_e^2(m_e^2 BR_{ee} + m_\mu^2 BR_{e\mu})} \quad (22)$$

respectively. As a result, we are able to indirectly determine the neutrino masses by measuring the quantities  $C_1$ ,  $C_2$ , and  $C_3$  in the tribimaximal limit of the neutrino mixings. One can see that the different terms in numerators of  $C_i$  ( $i=1-3$ ) are all of the same order of magnitude since the large differences in the branching ratios (see Table I) are compensated by the large differences of the masses of the charged leptons. The degree of accuracy in measuring  $C_i$  depends on the assumption of a sufficient number of like-sign leptons to be observed, so the sensitivity depends on the number of  $P^{\pm\pm}$  and the branching ratios of dilepton modes.

As we mentioned in Sec. III A, the branching ratios of the dilepton channels are sensitive to the values of  $\psi_{1,2}$  and  $m_1$  as given in Eq. (15). Thus,  $BR_{l_a^\pm l_b^\pm}$  will be very different from those shown in Table I with different values of  $\psi_{1,2}$  and  $m_1$ . However, we would use the ratio

$$\frac{BR_{ee}}{BR_{e\mu}} = \frac{1}{2} \frac{m_\mu^2 |m_{\nu_{ee}}|^2}{m_e^2 |m_{\nu_{e\mu}}|^2} = \frac{1}{2} \frac{m_\mu^2 \frac{5}{9} m_1^2 + \frac{4}{9} m_1 \sqrt{m_1^2 + \Delta m_{21}^2} \cos \psi_1 + \frac{1}{9} \Delta m_{21}^2}{m_e^2 \frac{2}{9} m_1^2 - \frac{2}{9} m_1 \sqrt{m_1^2 + \Delta m_{21}^2} \cos \psi_1 + \frac{1}{9} \Delta m_{21}^2} \quad (23)$$

to pin down the allowed parameter region as a function of the lightest neutrino mass  $m_1$ . This is demonstrated in Fig. 1 (right). Similar results of other ratios, such as  $\frac{BR_{ee}}{BR_{\mu\mu}}$ ,  $(\frac{BR_{ee}}{BR_{e\tau}}, \frac{BR_{e\mu}}{BR_{\mu\mu}})$ , and  $(\frac{BR_{e\mu}}{BR_{\mu\tau}}, \frac{BR_{\mu\mu}}{BR_{\mu\tau}})$  are displayed in Figs. 1 (left), 2 (left, right), and 3 (left, right), respectively. It is interesting to note that the branching ratio of  $P^{\pm\pm} \rightarrow e^\pm e^\pm$  reduces significantly in the small region around  $m_1 \sim 0.005$  eV due to the cancellations. In this region, the branching ratios of the rest dilepton modes will be enhanced (see Eq. (23), Figs. 1, and Fig. 2). Our relations of Eqs. (17)-(22) will be practically implemented for the lowest mass of the doubly charged Higgs scalars. If there is no cancellation in  $m_{\nu_{ee}}$ , we can still try to narrow down the parameter space of  $m_1$  and  $\psi_{1,2}$  by measuring the fractions of the dilepton modes. For instance, in the limit of  $m_1 \rightarrow 0$  the ratio of  $BR_{ee}/BR_{e\mu}$  becomes

$$\frac{BR_{ee}}{BR_{e\mu}} = \frac{m_\mu^2}{2m_e^2}. \quad (24)$$

On the other hand, if  $m_1$  is measured, the Majorana phase  $\psi_1$  can be extracted from the relation

$$\cos \psi_1 = \frac{(7C_4 - 3)m_1^2 + 2C_4 \Delta m_{21}^2}{(6 - 2C_4)m_1 \sqrt{m_1^2 + \Delta m_{21}^2}}, \quad (25)$$

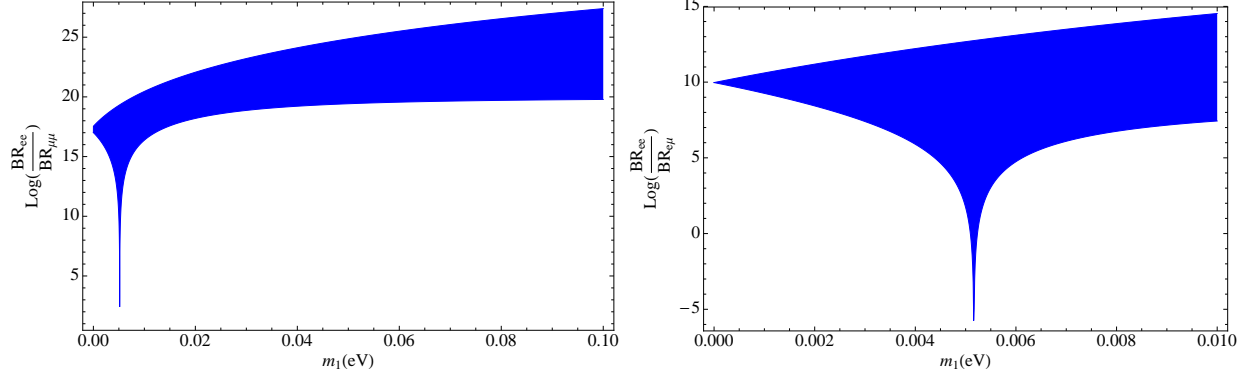


FIG. 1. Ratios of  $BR_{ee}/BR_{\mu\mu}$  (left) and  $BR_{ee}/BR_{e\mu}$  (right) versus the lightest neutrino mass  $m_1$  with scanning over the possible values of Majorana phases, where the shadow areas are the allowed regions.

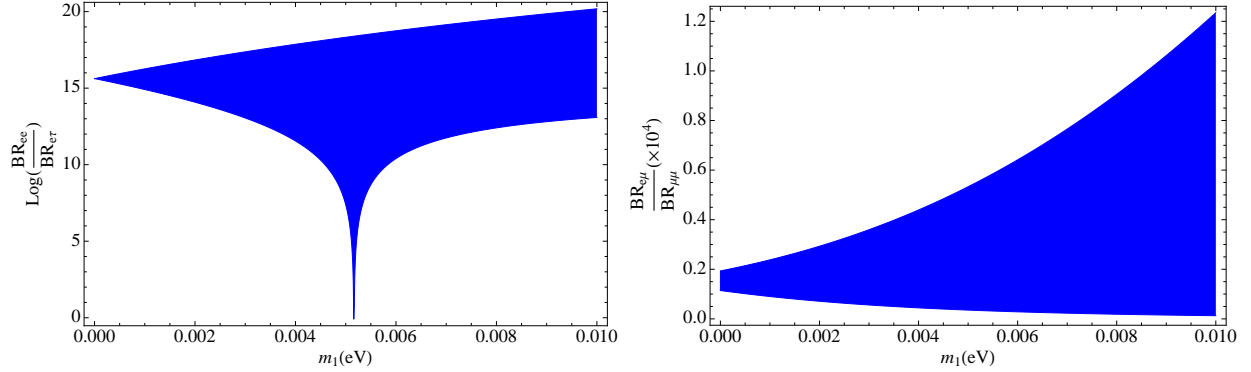


FIG. 2. Legend is the same as Fig. 1 but with the ratios of  $BR_{ee}/BR_{e\tau}$  (left) and  $BR_{e\mu}/BR_{\mu\mu}$  (right).

where  $C_4$  is expressed as

$$C_4 = \frac{2m_e^2 BR_{ee} - m_\mu^2 BR_{e\mu}}{2m_e^2 BR_{ee} + m_\mu^2 BR_{e\mu}}. \quad (26)$$

In Fig. 4, we plot the allowed region of  $\psi_1$  versus  $\frac{BR_{ee}-BR_{e\mu}}{BR_{ee}+BR_{e\mu}}$  and  $m_1$ . Note that the phase  $\psi_1$  becomes indefinite in Eq. (25) if we set  $m_1 \rightarrow 0$ . This is because when the lightest neutrino mass is zero, there is only one Majorana phase left, related to the relative phase of  $\psi_1 - \psi_2$  as shown in Eq. (15). In this limit, we obtain

$$\cos(\psi_1 - \psi_2) = \frac{\frac{2}{3}(\frac{BR_{\mu\mu}}{BR_{e\mu}})\frac{m_\mu^2}{m_e^2}\Delta m_{21}^2 - (\frac{13}{12}\Delta m_{21}^2 + \frac{3}{4}\Delta m_{32}^2)}{\Delta m_{21}\sqrt{\Delta m_{21}^2 + \Delta m_{32}^2}}. \quad (27)$$

The allowed region of  $\psi_1 - \psi_2$  for  $m_1 \rightarrow 0$  is displayed in Fig. 5, where we have taken the uncertainties of the mass differences from the solar and atmospheric data [2].

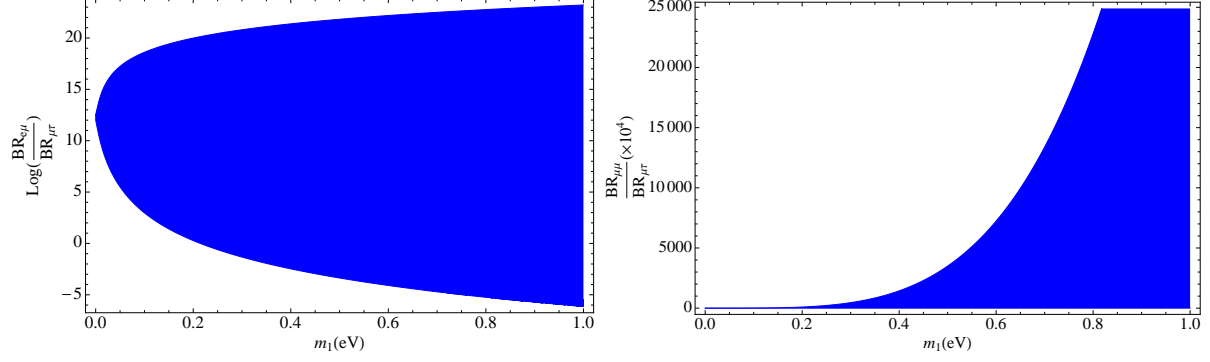


FIG. 3. Legend is the same as Fig. 1 but with the ratios of  $BR_{e\mu}/BR_{\mu\tau}$  (left) and  $BR_{\mu\mu}/BR_{\mu\tau}$  (right).

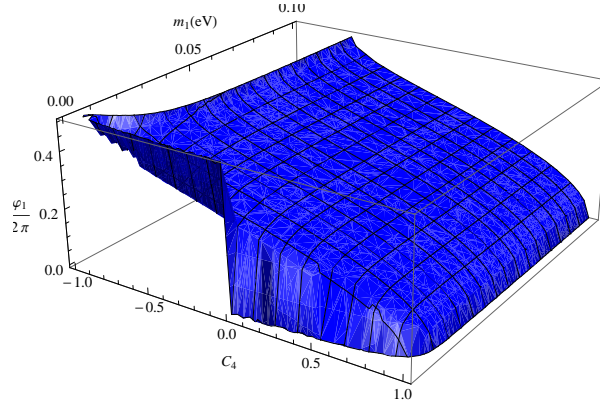


FIG. 4. The Majorana phase  $\psi_1$  in terms of  $C_4 = \frac{2m_e^2 BR_{ee} - m_\mu^2 BR_{e\mu}}{2m_e^2 BR_{ee} + m_\mu^2 BR_{e\mu}}$  and  $m_1$ .

#### IV. CONCLUSION

We have studied the close relationships between the neutrino masses and the same sign dilepton decays of the doubly charged Higgs scalars in the model in which the neutrinos are Majorana particles with their masses are generated radiatively at two-loop level. Since the dilepton modes in our model could be reachable at the LHC, it is natural to use their branching ratios to infer the neutrino masses. We have explicitly shown that in the limit of the tribimaximal mixings, the absolute scale of neutrino masses can be expressed in terms of  $C_i$  ( $i = 1-3$ ) based on the certain combinations of dilepton branching ratios. It is possible to determine the neutrino masses by just counting the events arising from the dilepton decays of the doubly charged Higgs in its lowest mass region. The allowed parameter space of the fractions among each dilepton branching ratio as the function of the lightest neutrino mass

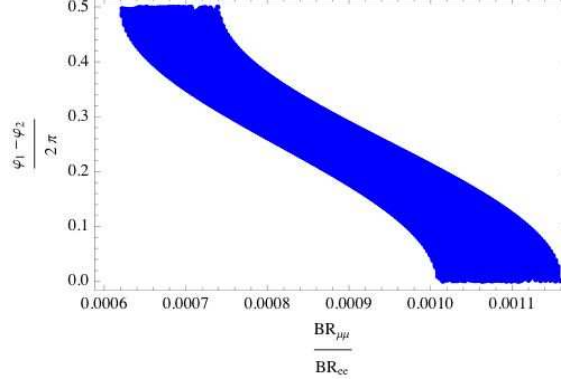


FIG. 5. The relative Majorana phase  $\psi_1 - \psi_2$  versus  $\frac{BR_{e\mu}}{BR_{\mu\mu}}$  for  $m_1 \rightarrow 0$ .

$m_1$  is presented. These relations combined with the data from other neutrino experiments may help to set a limit of  $m_1$  and Majorana phase  $\psi_{1,2}$  in the future.

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